# A De-Embedding Method to Characterize Differential Amplifiers Using Passive Baluns

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***Abstract*—This paper analyses the noise and gain measurement of microwave differential amplifiers using two passive baluns. A model of the baluns that includes potential losses and unbalances has been considered. This analysis allows to de-embed the actual performance of the differential device from the single-ended measurements of the two-port cascaded system and the baluns. The method has been validated with measured results from a fully-differential amplifier prototype.**

***Keywords- baluns; differential amplifiers; low-noise amplifiers, noise measurement.***

## I. INTRODUCTION

During the last decades, differential amplifier topologies have been commonly used in low frequency electronics. More recently, electronic designers have been working to increase the frequency range of such devices, in order to exploit their interesting properties at microwave frequencies. One potential application in the microwave range is the design of differential active antenna arrays for the *Square Kilometre Array* (SKA) radio-telescope [1]. In this array, Vivaldi-like antennas provide the incoming signals in balanced mode. Thus, the use of low-noise differential amplifiers directly connected to the balanced antennas seems a convenient solution, since it avoids the system noise increment introduced by a lossy balun. For this type of applications, accurate differential noise measurement methods are needed to characterize the noise performance of the low-noise amplifiers. However, one of the limitations of the differential amplifiers is their non-trivial characterization at microwave frequencies [2].

In the case of the scattering (S-) parameters, the use of the so-called mixed-mode parameters allows to characterize a differential device in terms of common/differential-mode excitations. These mixed-mode S-parameters can be easily obtained from the conventional ones [3] or directly measured in a multiport network analyzer [4]. However, the measurement of the noise figure of differential amplifiers is still a challenge, and several approaches have been proposed in the literature. In [5], a hot/cold method based on a differential load connected at the input of the amplifier is presented. This method requires to cool the load using liquid nitrogen, which may be not easily available. Furthermore, although resistive loads are easy to obtain, synthesizing arbitrary imaginary impedances is more difficult. In [6], a method based on single-ended measurements between pairs of ports is used to determine the noise and gain performance of a differential amplifier. The advantage is that this method only requires a conventional single-ended noise figure equipment. The problem comes for source-pull measurements, since it would require a balanced impedance tuner to respect the (anti-)symmetry of the circuit, which is not commonly available [7].

Nowadays, the most direct method to characterize a differential amplifier is using passive baluns. It allows to transform from the 4-port differential device into a 2-port equivalent circuit driven by differential-mode input/output excitations. Once the device is converted into a 2-port amplifier, it can be directly measured by using any conventional single-ended network or noise-figure analyzer, and even the noise circles can be obtained by using conventional source-pull tools [7][8]. The prize is that the baluns affect the measurement, and a method to de-embed the performance of the differential device from the measurement of the cascaded system is needed. De-embedding methods described in the literature assume a model of the amplifier with symmetrical losses and ideal 180º phase difference in the output ports, which can be very optimistic in practice [9].

This paper analyzes the measurement of differential amplifiers using baluns, but considering a more complete model of the baluns. In this case, potential losses and phase/amplitude unbalances in the branches of the balun are taken into account in the analysis. It includes the effect, not only of pure differential-mode signals as in [9], but the combined effect of common- and differential-mode signals. In practice, this analysis can be used to de-embed the noise figure and the gain of a differential amplifier from the measurement of the single-ended cascaded system. Finally, some measured results are presented to validate the theoretical study.

## II. THEORETICAL ANALYSIS

The measurement scheme that will be analyzed in this paper is shown in Fig. 1. The input and output baluns are denoted as $B1$ and $B2$, and the amplifier is denoted by $A$. In the ideal case, the input balun equally splits the signal coming from port 1 though ports 2 and 3, with 180º phase difference between both ports. The behavior of $B2$ is equivalent, but combining the signals. In such ideal case, the noise figure and the gain of the isolated differential amplifier (using differential excitations) is equal to the noise and gain of the cascaded system shown in Fig. 1 [9].

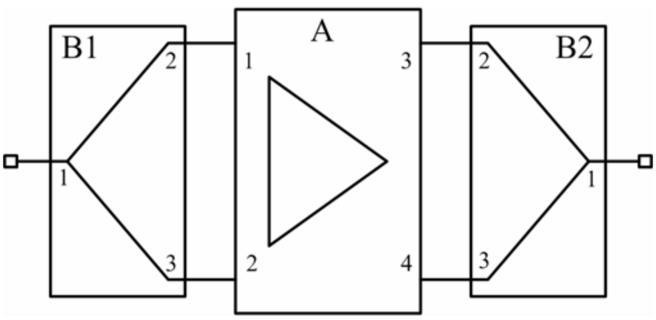


Fig. 1. Measurement setup.

There exist two types of differential amplifier: balanced and fully-differential [2] (Fig. 2). The first one is based on two independent single-ended amplifiers working in parallel. The second one, also known as differential pair, is based on two amplifiers with the sources of the transistors connected to a common current source. Their performance is equivalent when they are excited with differential signals. The difference is how they amplify the common-mode signals. In the balanced case, it equally amplifies the common- and differential-mode signals and, in the fully-differential case, it only amplifies differential-mode signals, since common-mode ones are mitigated by the structure. In terms of common-mode rejection ratio (CMRR), the first case provides CMRR=1, whereas the second case ideally provides CMRR=∞. Since we want to consider a model of the balun with potential unbalances, the propagation of both common- and differential-mode signals though the baluns is feasible, so a separate analysis for both types of amplifiers is needed.

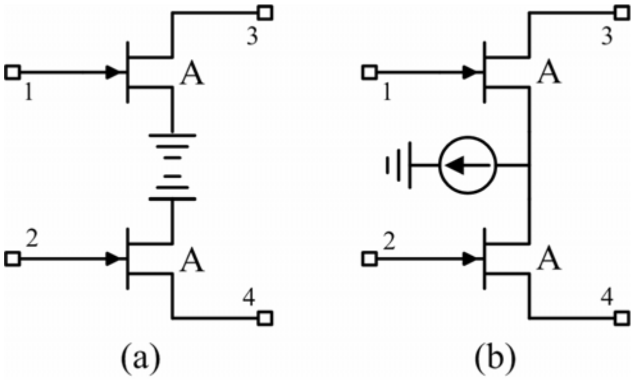


Fig. 2. Circuit schematic of a balanced (a) and a full-differential (b) amplifier.

## A. Balanced Amplifier

In the ideal case, the S-parameters of the balanced amplifier can be considered zero, except the transmission between ports 1-3 and 2-4, i.e.,

$$s_{31}^{A} = s_{42}^{A} = A \tag{1}$$

where $A$ is the gain factor of the amplifier. For the case of the baluns, it is assumed that all the S-parameters are zero (perfect impedance matching and output ports isolation), except the transmission coefficients between ports 1-2 and 1-3 (arbitrary losses and phase/magnitude unbalances), i.e.,

$$\begin{aligned} s_{12}^{B1} &= s_{21}^{B1},\ s_{13}^{B1} = s_{31}^{B1}, \\ s_{12}^{B2} &= s_{21}^{B2},\ s_{13}^{B2} = s_{31}^{B2} \end{aligned} \tag{2}$$

where super-index $B1$ and $B2$ indicates the input and output baluns respectively.

The noise factor of a two-port device can be written as

$$f = \frac{\text{Total noise power at the output}}{\text{Noise power at the output due to the source resistor}} \tag{3}$$

In this case, the noise power at the output port is generated by the contributions from the source resistor, the two baluns, and the amplifier. In the case of the input balun, it can be assumed that the two branches generate uncorrelated noise signal [9]. Since the noise factor of a passive device is equal to its losses, the noise spectral density at its output is $kT$ ($W^2$/Hz), where $k$ is the Boltzmann constant and $T$ is the system temperature. For the branches 1-2 and 1-3 of the input balun, these power densities propagates to the output through the amplifier and the output balun as

$$\begin{aligned} N_{1-2}^{B1,a} &= kT\,|\,s_{31}^{A}s_{12}^{B2}\,|^2 \\ N_{1-3}^{B1,a} &= kT\,|\,s_{42}^{A}s_{13}^{B2}\,|^2 \end{aligned} \tag{4}$$

where sub-index 1-2 and 1-3 indicates the corresponding branch of the balun; super-index $a$ refers to the balanced topology and super-indices $B1$, $A$, $B2$ denote the corresponding input balun or amplifier or output balun parameter. Terms in (4) contain the combined contribution of the source resistor and the input balun. For the balanced amplifier, it can be assumed that the noise generated by the two separated branches is uncorrelated. By definition, the noise power at the output of an amplifier is $kTf_AG_A$, where $f_A$ is the noise factor of the amplifier and $G_A$ is the power gain [9]. In order to calculate the contribution of the amplifier alone, the contribution from the source must be eliminated (i.e., $kTG_A$). Thus, the noise power of each stage of the balanced amplifier propagated to the output of the system is respectively

$$\begin{aligned} N_1^{A,a} &= (kTf_A\,|\,s_{31}^{A}\,|^2 - kT\,|\,s_{31}^{A}\,|^2)\,|\,s_{12}^{B2}\,|^2 \\ &= kT(f_A - 1)\,|\,s_{31}^{A}s_{12}^{B2}\,|^2 \\ N_2^{A,a} &= kT(f_A - 1)\,|\,s_{42}^{A}s_{13}^{B2}\,|^2 \end{aligned} \tag{5}$$

where sub-index 1 and 2 indicates the corresponding input port of the amplifier. The noise spectral density at the output of the balun $B2$ is $kT$ ($W^2$/Hz). Since two independent source resistors launch noise at port 2 and 3 of the output balun, the noise generated by the balun alone can be obtained as

$$\begin{aligned} N^{B2,a} &= kT - kT\,|\,s_{12}^{B2}\,|^2 - kT\,|\,s_{13}^{B2}\,|^2 \\ &= kT\left[1 - \left(\left|s_{12}^{B2}\right|^2 + \left|s_{13}^{B2}\right|^2\right)\right] \end{aligned} \tag{6}$$

Finally, the noise contribution from the source resistor at the output of the system is obtained as

$$N^{S,a} = kT\,|\,s_{21}^{B1}s_{31}^{A}s_{12}^{B2} + s_{31}^{B1}s_{42}^{A}s_{13}^{B2}\,|^2 \tag{7}$$

From the last expression, the power gain of the cascaded system when using a balanced amplifier can be extracted, i.e.,

$$G_{casc}^{(a)} = |\,s_{21}^{B1}s_{31}^{A}s_{12}^{B2} + s_{31}^{B1}s_{42}^{A}s_{13}^{B2}\,|^2 \tag{8}$$

Finally, the noise figure of the cascaded system can be obtained from (3)-(7) as

$$f_{casc}^{(a)} = \frac{N_{1-2}^{B1,a} + N_{1-3}^{B1,a} + N_{1}^{A,a} + N_{2}^{A,a} + N^{B2,a}}{N^{S,a}}$$
$$= \frac{f_A(|s_{31}^{A}s_{12}^{B2}|^2 + |s_{42}^{A}s_{13}^{B2}|^2) + 1 - (|s_{12}^{B2}|^2 + |s_{13}^{B2}|^2)}{|s_{21}^{B1}s_{31}^{A}s_{12}^{B2} + s_{31}^{B1}s_{42}^{A}s_{13}^{B2}|^2} \quad (9)$$

It can be seen that for the case of ideal baluns, i.e.,

$$s_{21}^{B1} = 1/\sqrt{2},\ s_{31}^{B1} = -1/\sqrt{2},$$
$$s_{12}^{B2} = 1/\sqrt{2},\ s_{13}^{B2} = -1/\sqrt{2} \quad (10)$$

the noise figure and the gain of the cascaded system is equal to the performance of the isolated amplifier, i.e.,

$$f_{casc} = f_A \text{, and } G_{casc} = A. \quad (11)$$

*B. Fully-Differential Amplifier*

Unlike in the previous case, the differential pair cannot be analyzed as two independent stages. Thus, in the ideal case, all the S-parameters of the amplifier are zero except the transmission parameters between the input ports and the output ports, i.e.,

$$s_{31}^{A} = s_{42}^{A} = A/2,$$
$$s_{32}^{A} = s_{41}^{A} = -A/2 \quad (12)$$

In this case, the noise power present in one of the inputs of the amplifier is propagated through the two outputs. Thus, the noise coming from one branch of the input balun is divided into two paths in the amplifier and combined in the output balun as

$$N_{1-2}^{B1,b} = kT\,|s_{31}^{A}s_{12}^{B2} + s_{41}^{A}s_{13}^{B2}|^2$$
$$N_{1-3}^{B1,b} = kT\,|s_{32}^{A}s_{12}^{B2} + s_{42}^{A}s_{13}^{B2}|^2 \quad (13)$$

where sub-index 1-2 and 1-3 indicates the corresponding branch of the balun. The noise contribution of the differential amplifier can be calculated assuming two independent noise sources in both input ports. Therefore, the noise launched by each source is propagated through the two outputs of the amplifier and finally combined by the output balun as

$$N_{1}^{A,b} = kT(f_A - 1)\,|s_{31}^{A}s_{12}^{B2} + s_{41}^{A}s_{13}^{B2}|^2$$
$$N_{2}^{A,b} = kT(f_A - 1)\,|s_{32}^{A}s_{12}^{B2} + s_{42}^{A}s_{13}^{B2}|^2 \quad (14)$$

where sub-index 1 and 2 indicates the corresponding input port of the amplifier. For the output balun, the noise coming from ports 2 and 3 cannot be considered uncorrelated in this case, since there are not two independent paths in the overall scheme. As the differential pair only propagates differential signals (common-mode gain is zero), the noise can be modeled as a differential source connected between ports 2 and 3. Then, the output balun can be interpreted as a two-port passive device, with a differential input port and a single-ended output port, whose transmission parameter $s_{sd21}$ can be obtained as

$$s_{sd21}^{B2} = (s_{12}^{B2} - s_{13}^{B2})/\sqrt{2} \quad (15)$$

Since the output noise power density is $kT$ ($W^2$/Hz), the contribution of the balun alone is obtained as

$$N^{B2,b} = kT(1 - 0.5\,|s_{12}^{B2} - s_{13}^{B2}|^2) \quad (16)$$

Finally, the noise contribution from the source resistor propagated to the output can be calculated from the combination of the different paths along which the source signal propagates, i.e.,

$$N^{S,b} = kT\,|(s_{21}^{B1}s_{31}^{A} + s_{31}^{B1}s_{32}^{A})s_{12}^{B2} + (s_{21}^{B1}s_{41}^{A} + s_{31}^{B1}s_{42}^{A})s_{13}^{B2}|^2 \quad (17)$$

The gain of the cascaded system in this case is directly obtained from (17) as

$$G_{casc}^{(b)} = |(s_{21}^{B1}s_{31}^{A} + s_{31}^{B1}s_{32}^{A})s_{12}^{B2} + (s_{21}^{B1}s_{41}^{A} + s_{31}^{B1}s_{42}^{A})s_{13}^{B2}|^2 \quad (18)$$

and the corresponding noise figure is

$$f_{casc}^{(b)} = \frac{N_{1-2}^{B1,b} + N_{1-3}^{B1,b} + N_{1}^{A,b} + N_{2}^{A,b} + N^{B2,b}}{N^{S,b}} =$$
$$= \frac{f_A(|s_{31}^{A}s_{12}^{B2} + s_{41}^{A}s_{13}^{B2}|^2 + |s_{32}^{A}s_{12}^{B2} + s_{42}^{A}s_{13}^{B2}|^2) + 1 - 0.5\,|s_{12}^{B2} - s_{13}^{B2}|^2}{|(s_{21}^{B1}s_{31}^{A} + s_{31}^{B1}s_{32}^{A})s_{12}^{B2} + (s_{21}^{B1}s_{41}^{A} + s_{31}^{B1}s_{42}^{A})s_{13}^{B2}|^2} \quad (19)$$

## III. Measured Results

This section shows the experimental measurements with a differential amplifier prototype. The circuit board of the fully-differential amplifier is shown in Fig. 3. It is based on two monolithic amplifiers, model ERA-5+ from *Minicircuits*. The sources of the two transistors have been connected to ground by means of a common series inductor, which acts as a current source.

The baluns used for the measurement are based on the well-known topology of a rat-race hybrid, which can provide equal amplitude splitting with anti-symmetrical phase response. The center frequency of the baluns is 2100 MHz. The measurement has been done between 1600 and 2600 MHz, which is the frequency range in which the reflection coefficients and isolation parameters of the baluns are acceptably low (i.e., lower than -15 dB). The measured parameters of the two baluns are shown in Fig. 4. It can be seen that the performance is closer to the ideal one (i.e., 3-dB transmission and 180º phase difference) at the center frequency, but it degrades in the edges of the band.

The characterization process of the differential amplifier is described below. Firstly, the S-parameters of the baluns were measured using a conventional network analyzer. Using the measurement setup depicted in Fig. 1, the gain and noise figure values of the cascaded system were measured using a conventional noise figure and gain equipment, which gives $G_{casc}$ and $f_{casc}$ values. From the measured values of $G_{casc}$ and the S-parameters of the baluns, and making use of expressions (12) and (18), the value of $A$ can be extracted. This value $A$ gives the de-embedded gain of the differential amplifier. It should be noticed that, in the case of a balanced amplifier, expressions (1) and (8) would be used instead. Lastly, from the measured value of $f_{casc}$, $A$ and the parameters of the baluns, and making use of (12) and (19), the value of the de-embedded noise figure of the amplifier $f_A$ can be obtained. In the case of the balanced amplifier, expressions (1) and (9) would be used instead.

The measured curves are represented in Fig. 5. The continuous lines represent the performance of the single-ended ERA-5+ amplifier. Since this amplifier has been used for the

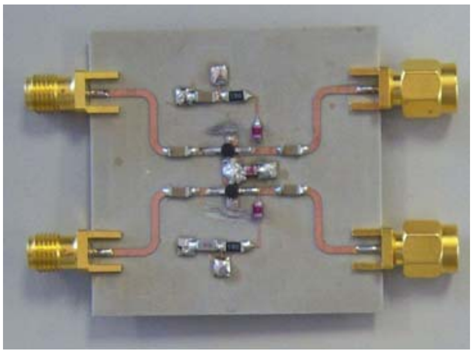

Fig. 3. Circuit board of the fully-differential amplifier.

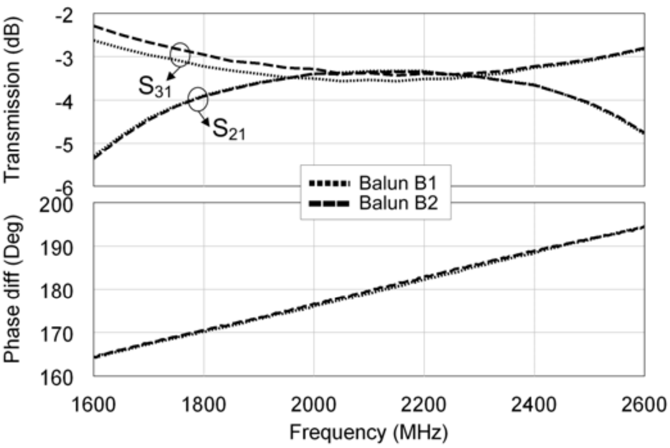


Fig. 4. Measured parameters of the baluns.

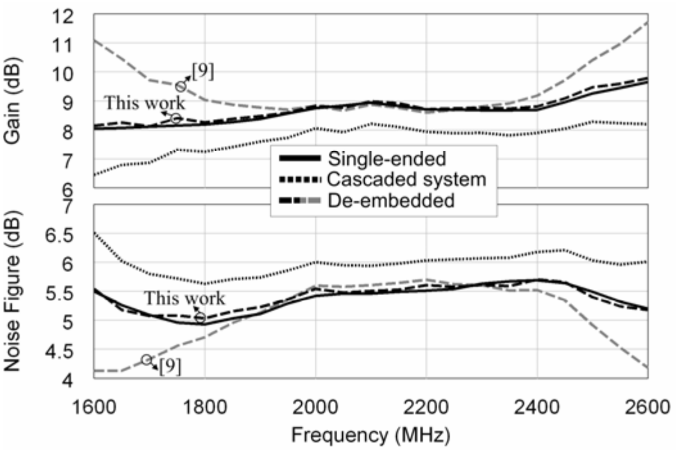


Fig. 5. Measured noise and gain results of the single-ended amplifier, the cascaded system with the amplifier and the baluns, and the de-embedded results using the present method and the method presented in [9].

design of the differential amplifier, it should ideally provide the same noise and gain results than the differential amplifier. This value is used as reference to measure the accuracy of the de-embedding process. The dotted lines represent the rough measurement of the cascaded system shown in Fig. 1. Finally, the dashed lines represent the de-embedded results, using the method described in this paper, and comparing with the result obtained with the method presented in [9]. It can be seen from the figure that the de-embedded curves obtained with the present method follows quite well the reference curve of the single-ended amplifier, even in the edge of the band in which the unbalances introduced by the baluns are higher. In the case of the method presented in [9], it presents similar results in the center of the band, but the error is very high in the edges of the band. It is logical since the method in [9] does not take into account the unbalances in the balun. Thus, the present method is inherently more accurate, since it considers the losses (as in [9]), but also the phase and amplitude unbalances of the baluns.

## IV. Conclusion

An improved de-embedding method to determine the noise figure of differential amplifiers from a measurement with baluns has been presented. A general model of the balun, including potential losses and unbalances has been considered for the analysis, which in practice allows to improve the reliability of the results. The method has been validated from the measurement of a differential amplifier prototype, obtaining good results.

## Acknowledgment


This work has been partially granted by the Spanish MEC under Project TEC2009-14525-C02-01, and by the European Community Framework Programme 6, Square Kilometre Array Design Studies (SKADS), contract no 011938.


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